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# FREE OSCILLATIONS OF THE EARTH OBSERVED BY A GALITZIN SEISMOGRAPH AT ABUYAMA, JAPAN

By

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**Abstract**

The great Chilean earthquake of May 22, 1960 was observed with a Galitzin seismograph installed at Abuyama Seismological Observatory of Kyoto University, Japan. Only a vertical component of record was used in the present investigation.

The record was read at every minute during a period of about 5.5 days beginning on May 19, 11 h 00 m, 1960 (UT). The whole reading was divided into five intervals in order to investigate the free oscillations of the earth excited by both the main shock and one of the large foreshocks of the Chilean earthquake. A low-cut filter was applied twice to all of the reading values, and then a Fourier analysis was made for every interval. It has come to be established that the free oscillations of the earth—spheroidal oscillations—could be detected by the present investigation and that their periods were in excellent agreement with periods obtained at other observation stations for the same earthquake and also with those predicted by theoretical investigations.

An analysis was also made using data reread at 15 second intervals in order to investigate a fine spectral structure for lower modes of the spheroidal oscillations, and a splitting was found in spectral peaks corresponding to modes ${}_0S_2$, ${}_0S_3$ and ${}_0S_4$.

## 1. Introduction

Free oscillations of the earth have theoretically being studied since the latter part of the nineteenth century, while they were investigated for records obtained at the time of the great Chilean earthquake of May 22, 1960. The earth's free oscillations excited by this earthquake were detected with various instruments at several observation stations over the world (Benioff et al. [1961], Ness et al. [1961], Alsop et al. [1961], Bogert [1961], Buchheim et al. [1961], Bolt et al. [1962], Nakagawa et al. [1964]), and obtained periods corresponding to the earth's free oscillations were in excellent agreement with periods theoretically predicted on the basis of an earth's model after Gutenberg and others. In Japan, the free oscillations of the earth were observed with an

Askania gravimeter working for the purpose of observing the earth tides at Kyoto (Nishimura et al. [1961], Takeuchi et al. [1962]), and periods obtained were in good agreement with those obtained at other observation stations, as well as theoretical periods.

A trial to detect free oscillations of the earth using the record (vertical component) obtained with a Galitzin seismograph at Abuyama Seismological Observatory of Kyoto University, at that time, is described in the present article. Several large earthquakes were followed successively by the great Chilean earthquake before and after the main shock. A similar investigation is also made for one of them.

## 2. Data and method of analysis

The position of observation station, characteristics of the instrument and epicentral data of the earthquakes under consideration are shown in Table 1.

Table 1. Description of the observation station, instrument and epicentral data

| Observation station | | | | |
|---|---|---|---|---|
| | Longitude | Latitude | Altitude | Depth |
| Abuyama | 135°34′22″E | 34°51′24″N | 220 m | 2 m |

| Instrument used | | | |
|---|---|---|---|
| | Constant | | Recording speed |
| Galitzin seismograph (vertical) | $T_p = 8.0$ sec<br>$T_g = 81.0$ sec<br>$V_{max} = 860$ | $h_p = 1.4$<br>$h_g = 1.0$<br>$\sigma = 0.1$ | 20 mm/min |

| Epicentral data | | | |
|---|---|---|---|
| | Origin time | Epicentre | Magnitude |
| Foreshock | May 21, 10:02:50, 1960 (UT) | $37\frac{1}{2}$° S, $73\frac{1}{2}$° W | $7\frac{1}{4} \sim 7\frac{1}{2}$ |
| Main shock | May 22, 19:11:20, 1960 (UT) | 38° S, $73\frac{1}{2}$° W | $8\frac{1}{4} \sim 8\frac{1}{2}$ |

Reading of the original record was made at every minute during a period of about 5.5 days from May 19, 11 h 00 m, 1960 to May 25, 00 h 19 m, 1960 (UT). It was made again at every 15 seconds for a whole day of May 23, 1960 (UT). An accuracy of the reading was up to 0.1 mm on the recording paper. The whole reading period was divided into five intervals. They are hereinafter referred to as intervals I, II, III, IV and V, respectively, and their details are shown in both Table 2 and Fig. 1. The values read at both every minute and every 15 seconds are available for the interval IV, while the values read at every minute are available for other intervals.

Table 2. Different intervals adopted in analysis

| | First time | Last time | Origin time of analysis | Number of readings |
|---|---|---|---|---|
| Interval I | May 19, 11:00 | May 20, 10:59 | May 19, 22:59 | 1440 |
| Interval II | May 19, 13:00 | May 20, 10:59 | May 19, 23:59 | 1320 |
| Interval III | May 21, 12:30 | May 22, 10:29 | May 21, 23:29 | 1320 |
| Interval IV | May 23, 00:00 | May 23, 23:59 | May 23, 11:59 | 1440 |
| Interval V | May 24, 00:00 | May 24, 23:59 | May 24, 11:59 | 1440 |

(Time in UT)

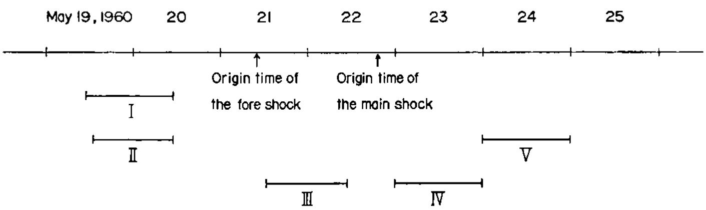


Fig. 1. Different intervals adopted in analysis.

Speaking in more detail about the intervals:

Interval I : Quiet 24 hours before when the foreshock was occurred.

Interval II : 22 hours including in the interval I ; that is, quiet 22 hours before when the foreshock was occurred. Last time of this interval was the same as that of the interval I.

Interval III : 22 hours just after when the foreshock was occurred. The earth's free oscillations excited by the foreshock were expected to be recorded in this interval, if they were existing.

Interval IV : 24 hours just after when the main shock (the great Chilean earthquake) was occurred. Free oscillations of the earth excited by the main shock were expected to be recorded in this interval, if they were existing.

Interval V : 24 hours following the interval IV. The earth's free oscillations were also expected to be recorded in this interval, if they were existing, but their amplitudes should be small in comparison with those for the interval IV.

When free oscillations of the earth are discussed, periods extending several tens of minutes to several minutes must be taken into account. In order to remove waves with periods longer than 2 hours, a low-cut filter was applied twice to all of the values read at every minute. 119 reading values at each end of the intervals disappeared in each filtering process and consequently the total numbers of the values amounted to

$$1440-119\times2\times2=964 \quad \text{for the intervals I, IV and V.}$$

$$1320-119\times2\times2=844 \quad \text{for the intervals II and III.}$$

The values thus obtained were analyzed by Fourier's method.

The values read at every 15 seconds for the interval IV were analyzed by Fourier transform method, without applying any low-cut filter, the number of the values used being 5760.

## 3. Results of the analysis

Results of the Fourier analysis are shown in Figs. 2 to 6. Figs. 2 to 6 correspond to the intervals I to V, respectively. The values read at every minute were used in the analysis. Results of the analysis by the Fourier transform method are also shown in Figs. 7 to 10. The values read at every 15 seconds were used in this analysis. Theoretical periods of the earth's spheroidal oscillations calculated for the earth's model after Gutenberg-Bullen A are also shown in Figs. 4 to 10.

## 4. Discussion and conclusion

As Fig. 2 is power spectrum of the record for the quiet period (interval I) before the foreshock, any remarkable peak should not originally be seen in

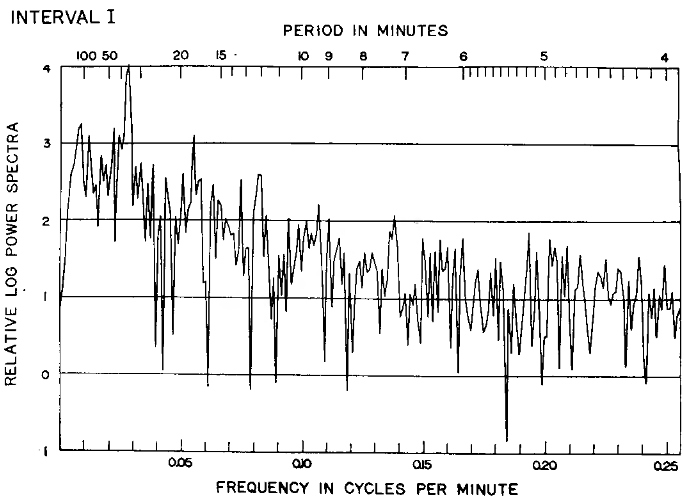


Fig. 2. Power spectrum corresponding to the interval I.

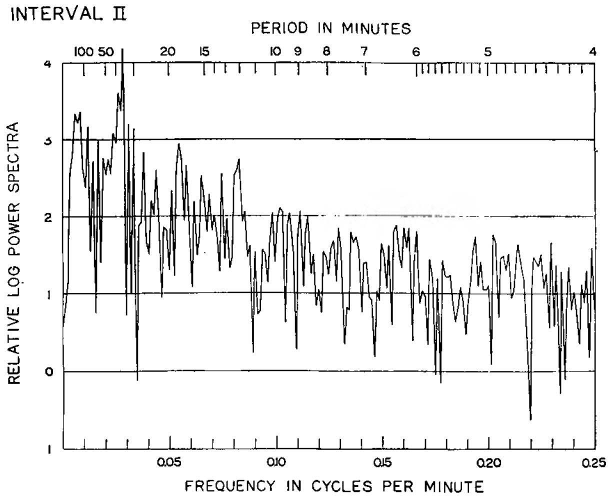


Fig. 3. Power spectrum corresponding to the interval II.

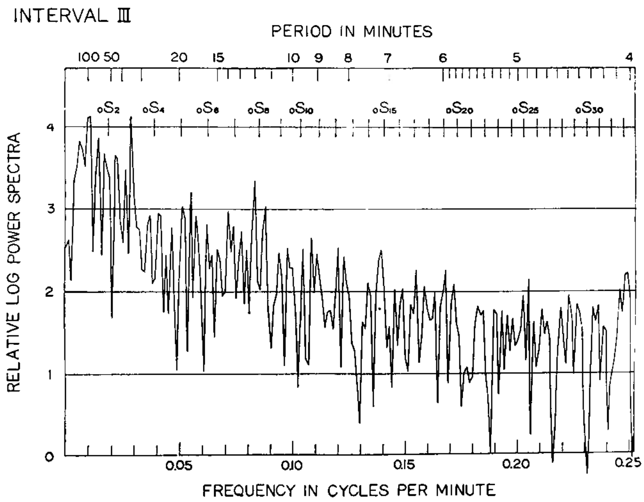


Fig. 4. Power spectrum corresponding to the interval III.

the spectrum. But, several strong peaks are actually found in the spectrum, as shown in Fig. 2. Their periods are as follows :

35.6, 18.2, 12.2, 9.34, 7.23, 6.01 and 5.17 minutes.

Taking the rotating period (about 36.3 minutes) of the recording drum of the seismograph into consideration, these periods recognized in Fig. 2 are due to the rotating period of the record. Except these strong peaks, Fig. 2 shows noise level of the record.

On the contrary, Fig. 5 shows power spectrum of the record for the

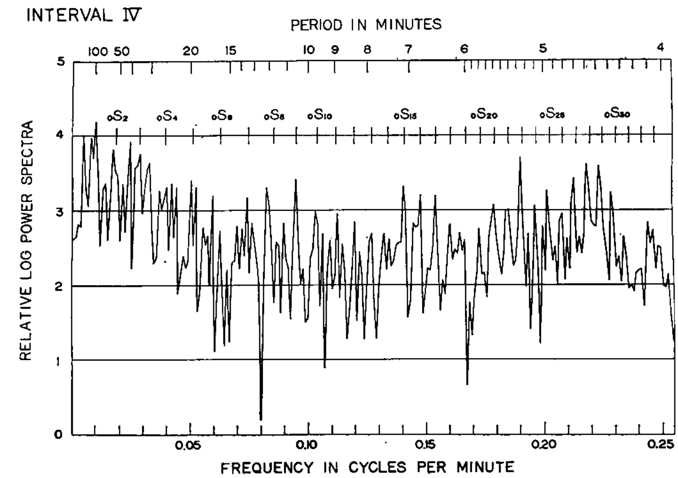


Fig. 5. Power spectrum corresponding to the interval IV.

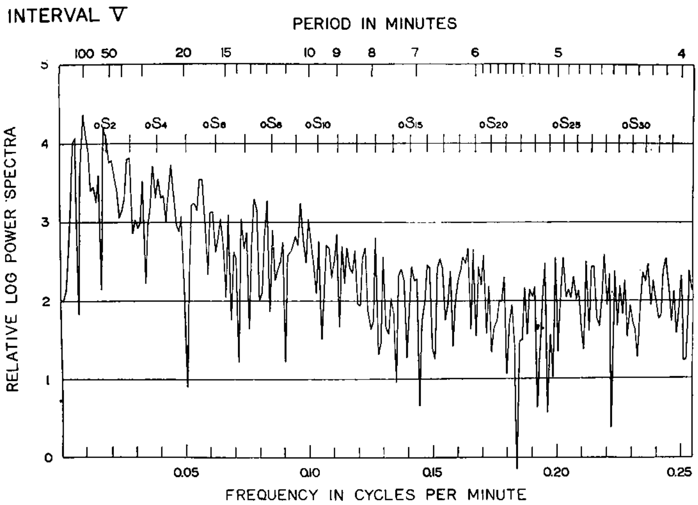


Fig. 6. Power spectrum corresponding to the interval V.

disturbed period (interval IV) after the main shock of the great Chilean earthquake. Many remarkable peaks are found in the spectrum of this figure in comparison with that of Fig. 2 and it is therefore perspicuous that oscillations corresponding to these peaks in Fig. 5 were excited by the main shock. Theoretical periods calculated for the Gutenberg-Bullen A earth's model are also shown in Fig. 5. As can easily be seen from Fig. 5, the positions of the peaks obtained by the present analysis are in good agreement with those theoretically predicted.

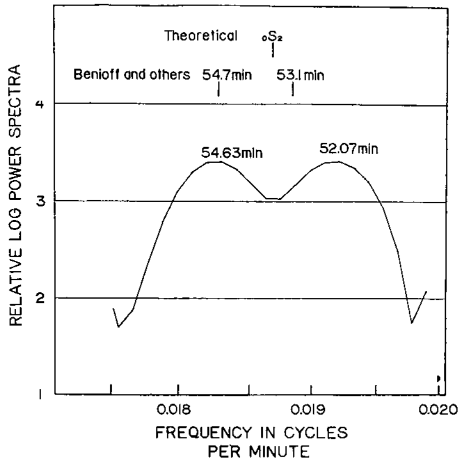


Fig. 7. Fine structure of power spectrum around $_0S_2$ (interval IV).

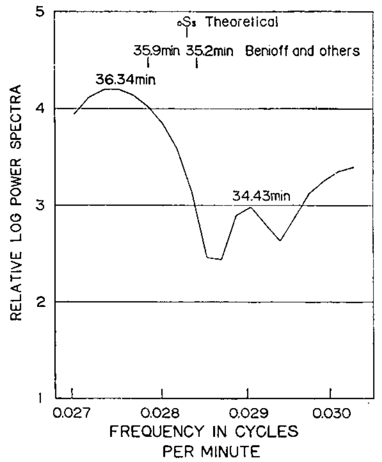


Fig. 8. Fine structure of power spectrum around $_0S_3$ (interval IV).

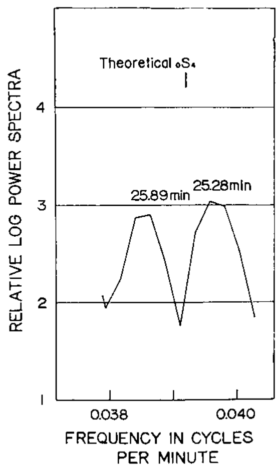


Fig. 9. Fine structure of power spectrum around $_0S_4$ (interval IV).

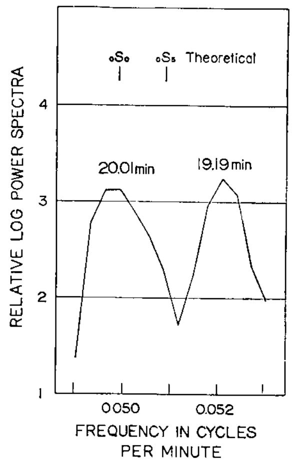


Fig. 10. Fine structure of power spectrum around $_0S_5$ (interval IV).

Several large earthquakes were successively occurred before and after the main shock. In order to investigate whether free oscillations of the earth had been excited by such earthquakes or not, similar analysis was made for the earthquake occurred at May 21, 10 h 02 m 50 s, 1960 (UT). 1320 reading values were used in the analysis for the interval III. For the comparison purpose, the same number of reading values was selected from the undisturbed interval I. Power spectrum of the record for the undisturbed period (interval II) before the foreshock and that for the disturbed period (interval III) after the foreshock are shown in Figs. 3 and 4, respectively. Fig. 3 shows noise level of the record, notwithstanding the following periods can be picked up from this figure as spectral peaks,

35.1, 18.3, 12.0, 8.96, 7.39, 6.01 and 5.17 minutes.

These are due to the rotating period of the recording drum.

If the earth's free oscillations were excited by the foreshock, spectral peaks corresponding to them should be recognized in Fig. 4. Examining in detail the spectrum of Fig. 4 compared with that of Fig. 3, some of the spectral peaks are found commonly in both Figs. 3 and 4, while many other peaks are also recognized only in the spectrum of Fig. 4. The spectral peaks found commonly in Figs. 3 and 4 are, as described above, due to the rotation of the recording drum and those only in Fig. 4 can be regarded as the earth's free oscillations excited by the foreshock. The spectral peaks obtained for the interval III (Fig. 4) are neither so remarkable as those for the interval IV (Fig. 5) nor always coincide with positions for the theoretical ones. Taking magnitude of the earthquakes into consideration, it is natural that the free oscillations of the earth excited by the foreshock are smaller than those by the main shock. It should be significant that similar investigation will, for the future, be made for records of earthquakes with magnitude of about $7\frac{1}{2}$, because a great earthquake whose magnitude is more than 8 will be of rare occurrence.

In Fig. 6 is shown power spectrum of the record for the interval V. The spectral peaks shown in Fig. 6 are not so remarkable as those in Fig. 5, owing to rapid attenuation of the seismograph.

Periods corresponding to the spectral peaks obtained by the present analyses are shown in Table 3. In this table, theoretical periods calculated for the earth's model after Gutenberg-Bullen A are also shown.

In case of large wave number $n$, phase velocity $C$ of surface waves is expressed by the following formula,

$$C=\frac{2\pi a}{(n+0.5)\,T},$$

where $T$ and $a$ are period for each mode and radius of the earth, respectively. The surface waves corresponding with spheroidal oscillations are Rayleigh waves in the case of the earth's free oscillations. Phase velocities were calculated using the periods for each mode of the earth's free oscillations obtained for the interval IV. They are shown in Fig. 11. As can easily be seen from Fig. 11, the phase velocities thus obtained are slightly smaller than those theoretically calculated for the Gutenberg-Bullen A earth's model.

Table 3. Observed and theoretical periods of free oscillations of the earth

| | Theoretical periods (Gut.-Bul. A) | Periods observed at Abuyama | | |
|---|---|---|---|---|
| | | Interval III | Interval IV | Interval V |
| ${}_0S_2$ | 53.50 | 56.1 | 53.4 | 53.4 |
| ${}_0S_3$ | 35.32 | 35.1 | 35.6 | 35.6 |
| ${}_0S_4$ | 25.53 | 24.8 | 25.3 | 25.3 |
| ${}_0S_5$ | 19.65 | 19.6 | 19.2 | |
| ${}_0S_6$ | 15.92 | 16.2 | 16.0 | 16.3 |
| ${}_0S_7$ | 13.43 | 13.6 | 13.5 | 13.7 |
| ${}_0S_8$ | 11.73 | 12.0 | 12.2 | 12.0 |
| ${}_0S_9$ | 10.53 | 10.7 | 10.6 | 10.6 |
| ${}_0S_{10}$ | 9.650 | 9.57 | 9.72 | 9.62 |
| ${}_0S_{11}$ | 8.950 | 8.96 | 8.91 | 8.99 |
| ${}_0S_{12}$ | 8.383 | 8.34 | 8.37 | 8.44 |
| ${}_0S_{13}$ | 7.887 | | 7.89 | 7.89 |
| ${}_0S_{14}$ | 7.475 | 7.45 | 7.46 | |
| ${}_0S_{15}$ | 7.108 | 7.20 | 7.13 | 7.07 |
| ${}_0S_{16}$ | 6.782 | 6.90 | 6.77 | 6.77 |
| ${}_0S_{17}$ | 6.492 | 6.48 | 6.50 | 6.54 |
| ${}_0S_{18}$ | 6.230 | | 6.25 | |
| ${}_0S_{19}$ | 5.993 | 5.97 | 6.09 | 6.09 |
| ${}_0S_{20}$ | 5.778 | | 5.80 | 5.87 |
| ${}_0S_{21}$ | 5.582 | | 5.59 | 5.59 |
| ${}_0S_{22}$ | 5.402 | 5.43 | 5.40 | |
| ${}_0S_{23}$ | 5.235 | 5.26 | 5.26 | 5.29 |
| ${}_0S_{24}$ | 5.080 | 5.07 | 5.09 | 5.12 |
| ${}_0S_{25}$ | 4.937 | 4.95 | 4.96 | 4.93 |
| ${}_0S_{26}$ | 4.802 | | 4.81 | 4.83 |
| ${}_0S_{27}$ | 4.675 | | 4.69 | 4.72 |
| ${}_0S_{28}$ | 4.557 | 4.58 | 4.58 | 4.56 |
| ${}_0S_{29}$ | 4.445 | 4.43 | 4.47 | 4.47 |
| ${}_0S_{30}$ | 4.340 | | 4.37 | 4.39 |
| ${}_0S_{31}$ | 4.238 | 4.25 | 4.28 | 4.26 |
| ${}_0S_{32}$ | 4.143 | | 4.15 | 4.18 |
| ${}_0S_{33}$ | 4.052 | 4.03 | 4.09 | 4.09 |

(Unit in minutes)

Magnification curve of the seismograph is shown in Fig. 12. An amplitude of the vertical motion of ground corresponding to the spheroidal oscillations in the interval IV is calculated as Table 4.

As shown in Table 4, the amplitude of the vertical motion for the mode ${}_0S_2$ is 17.2 mm. On the other hand, its value has been calculated to be 5.2 mm from data obtained with an Askania gravimeter at Kyoto. The amplitude for the

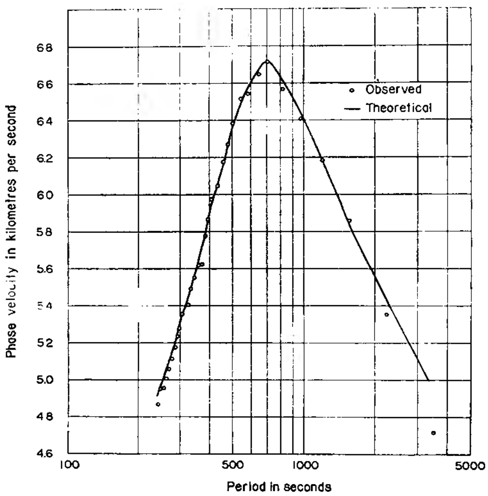


Fig. 11. Observed and theoretical (Gutenberg-Bullen A) phase velocity.

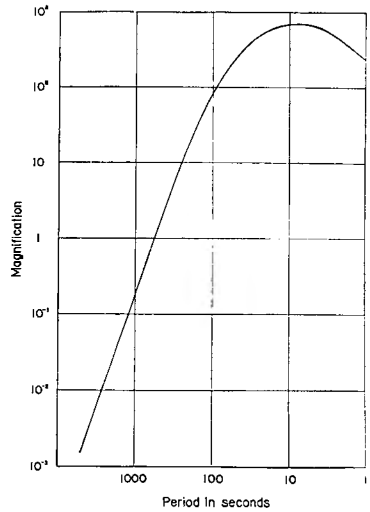


Fig. 12. Magnification curve of the Galitzin seismograph.

Table 4. Amplitude of the vertical motion of the ground excited by the great Chilean earthquake

| | Observed period (minutes) | Amplitude (millimetres) |
|---|---|---|
| ${}_0S_2$ | 53.4 | 17.2 |
| ${}_0S_3$ | 35.6 | 10.3 |
| ${}_0S_4$ | 25.3 | 2.47 |
| ${}_0S_5$ | 19.2 | 0.998 |
| ${}_0S_{15}$ | 7.13 | 0.505 |
| ${}_0S_{21}$ | 5.59 | 0.191 |
| ${}_0S_{29}$ | 4.47 | 0.192 |

mode ${}_0S_2$ obtained by the present analysis is therefore about three times larger than that obtained by the Askania gravimeter. $Q$ values for higher modes ($n=9\sim30$) calculated from amplitude decay for the intervals IV and V are hundreds. The $Q$ values obtained are much larger than those obtained at some observation stations at the time of the same earthquake.

Generally speaking, spectral peaks of higher modes ($n=9\sim30$) are considerably clear in comparison with those of lower modes, as can easily be seen from Fig. 5. This is due to the fact that the number of waves of higher modes included in a certain interval is much larger than that of lower modes.

In the case when the reading from the record was done at every minute, the position of the spectral peaks corresponding to lower modes is unreliable because of roughness in period difference of adjoining wave numbers. The record of the interval IV was then reread at every 15 seconds and the reading values obtained were analyzed by Fourier transform method, without applying any low-cut filter. Results of the analysis near the modes ${}_0S_2$, ${}_0S_3$, ${}_0S_4$ and ${}_0S_5$ are shown in Figs. 7 to 10, respectively. Spectral splitting is recognized for the modes ${}_0S_2$, ${}_0S_3$ and ${}_0S_4$, as shown in Figs. 7 to 9, although the splitting around a period corresponding to the mode ${}_0S_3$ is less clear than that for the modes ${}_0S_2$ and ${}_0S_4$. Such a separation in spectrum is also recognized in Fig. 10. Fig. 10 has maxima at 20.0 and 19.2 minutes. The former may be the mode ${}_0S_0$ and the latter ${}_0S_5$. The spectral splitting has been observed in America at the time of the same earthquake (Benioff et al. [1961], Ness et al. [1961]). Results for the modes ${}_0S_2$ and ${}_0S_3$ obtained by Benioff and others are also shown in Figs. 7 and 8, respectively. Obtained periods of the spectral splitting are shown in Table 5. The periods of the splitting obtained by the present investigation are slightly shorter than those obtained by Benioff and others.

Table 5. Observed periods of the spectral splitting

| | Observed period | Benioff et al. |
|---|---|---|
| ${}_0S_2$ | 54.6 and 52.1 | 54.7 and 53.1 |
| ${}_0S_3$ | 36.3 and 34.4 | 35.9 and 35.2 |
| ${}_0S_4$ | 25.9 and 25.3 | |

(Unit in minutes)

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